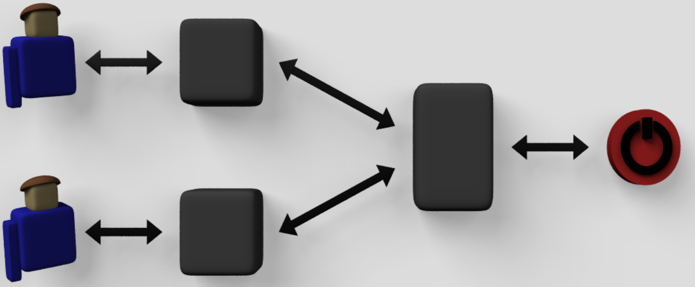
On Classes of Distributed Petri Nets

Jens-Wolfhard Schicke-Uffmann

2018-04-11

Motivating Example

Consider a network service with (wlog. 2) users:



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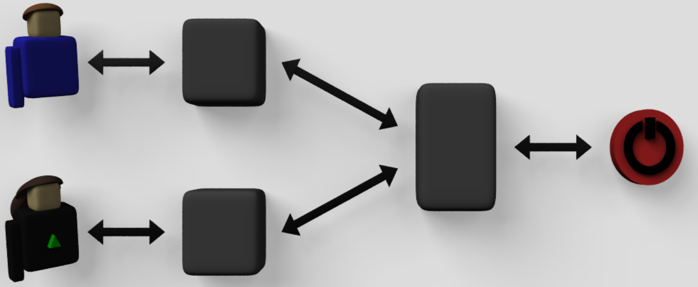

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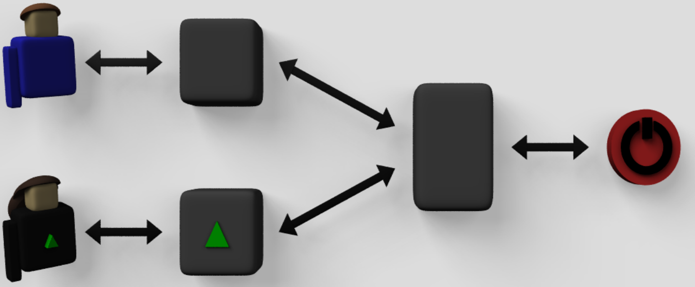

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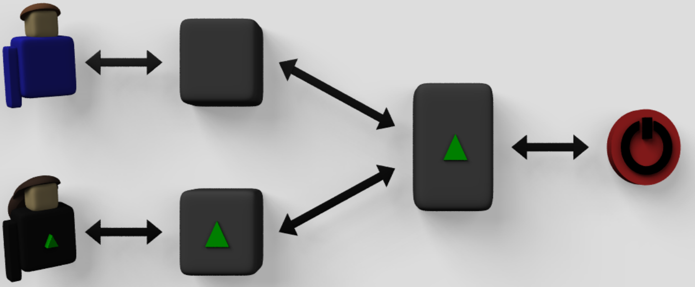

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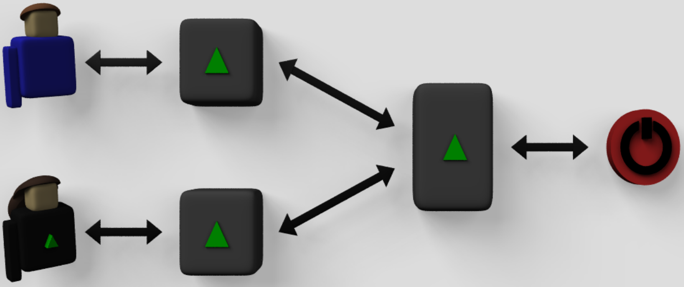

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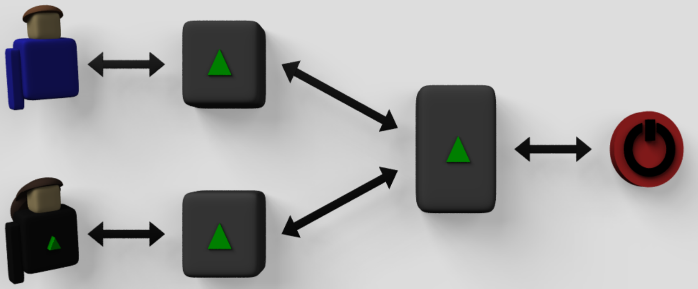

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Abstract view: Hacker uses information flow + magic.

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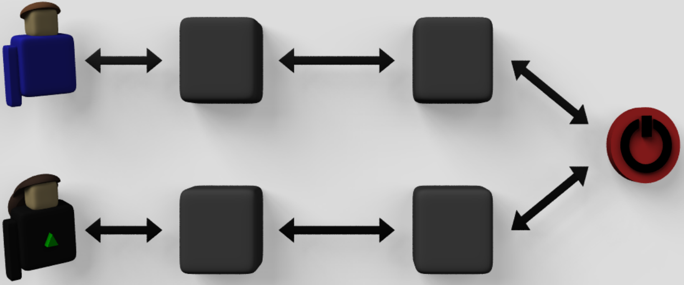
Separate information flows!



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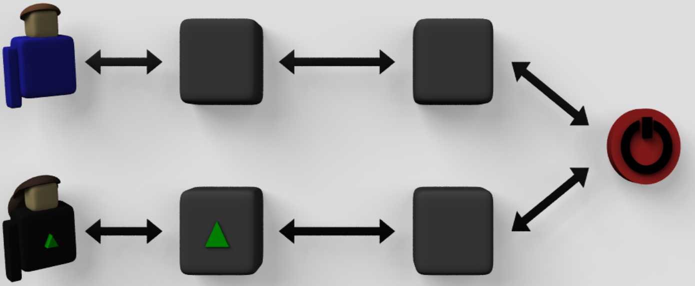

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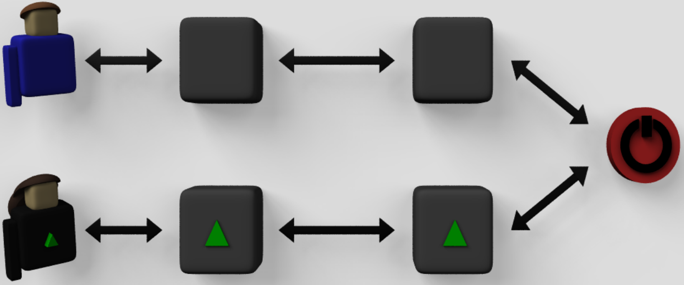

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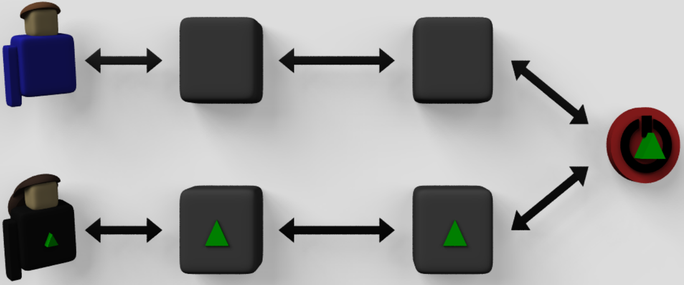

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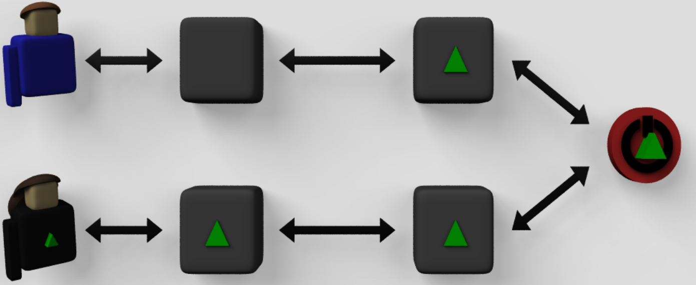

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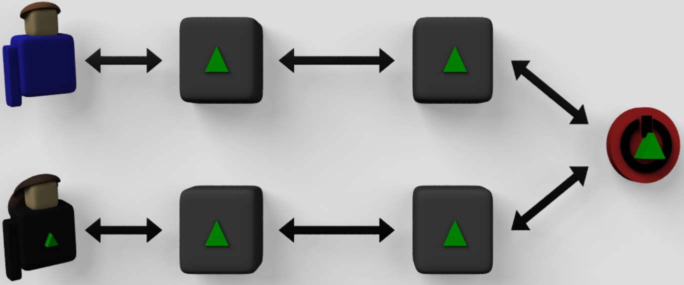

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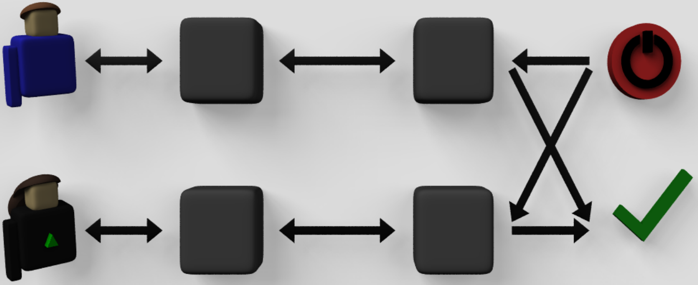

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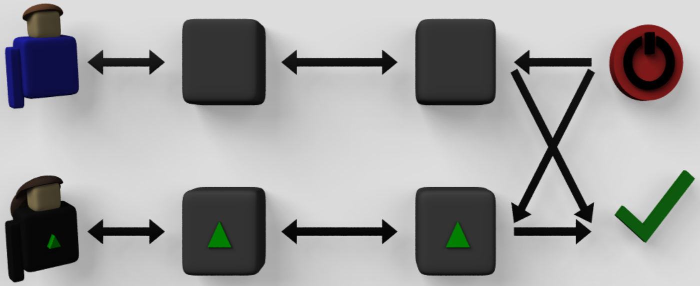

Are We Just Too Stupid?

- Maybe there is a better way?

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- Big Data?

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- Maybe there is a better way?
- Paxos?

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- Maybe there is a better way?
- Blockchain?

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- Maybe there is a better way?
- Edge Computing?

Are We Just Too Stupid?

- Maybe there is a better way?
- Use infinitely many computers?
- What exactly is an acceptable solution here?

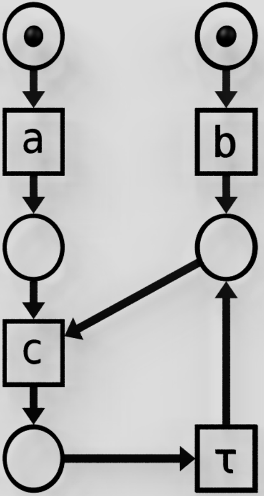
Formal Model

Abstract as much as possible. We need:

- (Parts of) the system can be in different states
- (Parts of) the system can do various things
- The parts of the system are spatially distributed
- Parts of the system send each other information

Petri nets

A *Petri net* is a tuple
$N = (S, T, F, M_0, \ell)$ with



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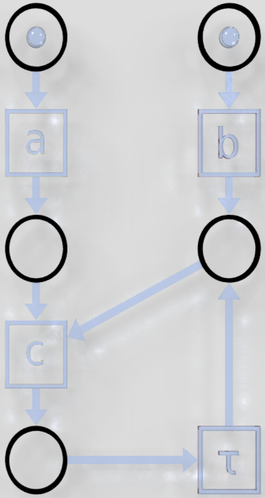

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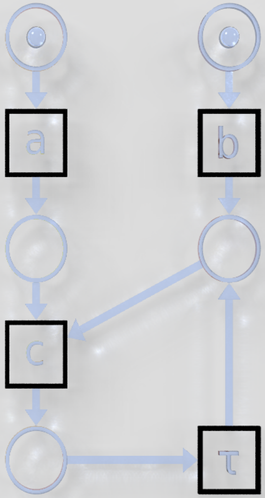

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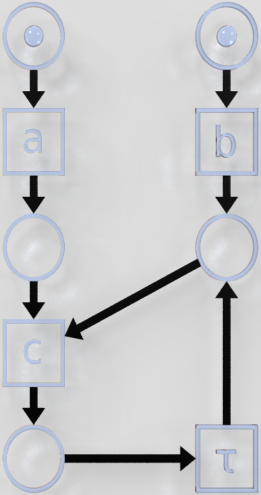

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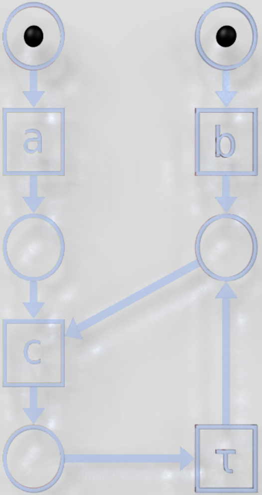

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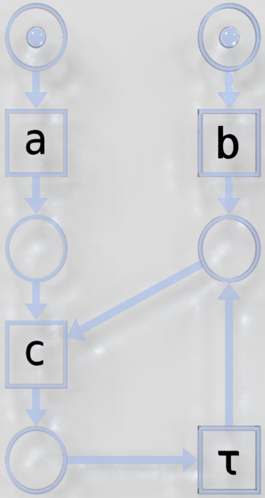
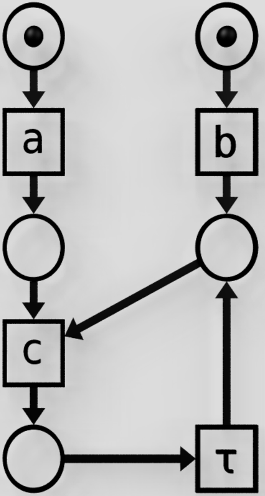
- $\ell : T \to \mathrm{Act} \cup \{\tau\}$ a *labelling function*.



Dynamic Behaviour – Firing Rule

Let $N = (S, T, F, M_0, \ell)$ be a net.

- A multiset $M \in \mathbb{N}^S$ is a *marking* of N.
- $t \in T$ is *enabled* if ${}^\bullet t \leq M$.
- A nonempty, finite $G \in \mathbb{N}^T$ is a *step* from M to marking M' iff ${}^\bullet G \leq M$ and $M' = M - {}^\bullet G + G^\bullet$.
- $[M_0\rangle$ denotes the set of *reachable* markings.
- If $[M_0\rangle \subseteq \{0, 1\}^S$ the net is *1-safe*.



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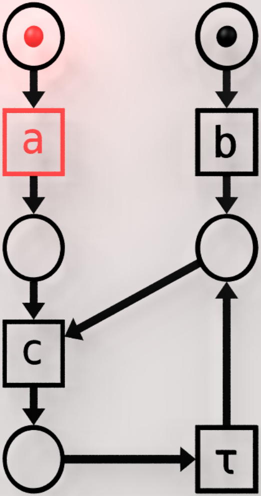

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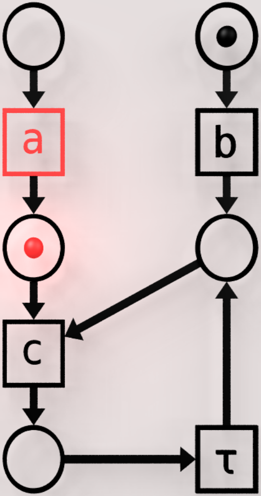

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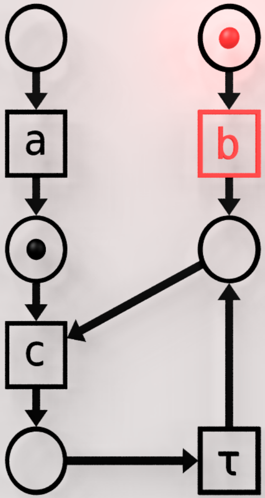

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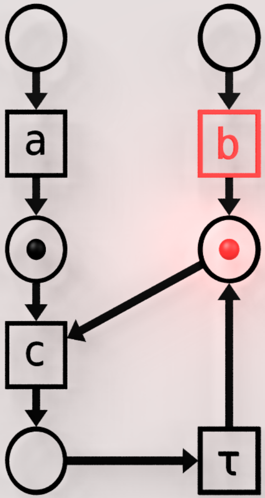

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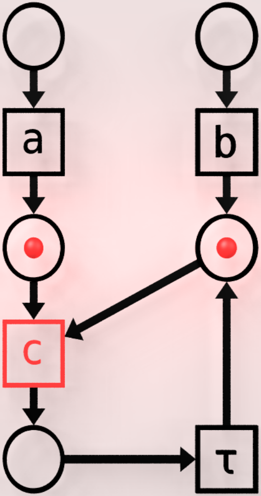

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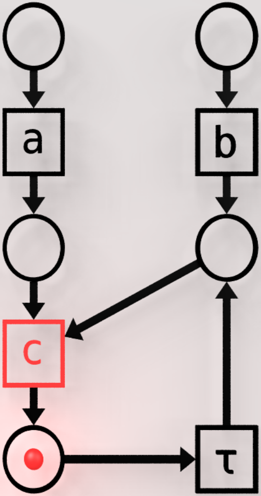

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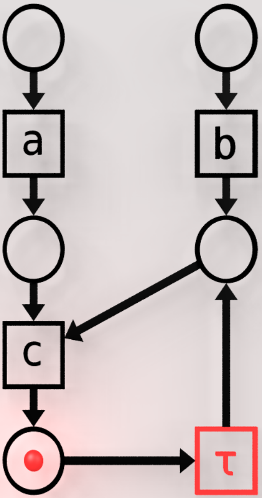

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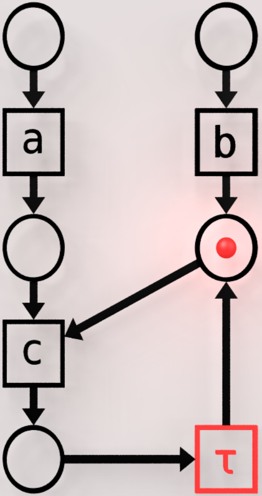

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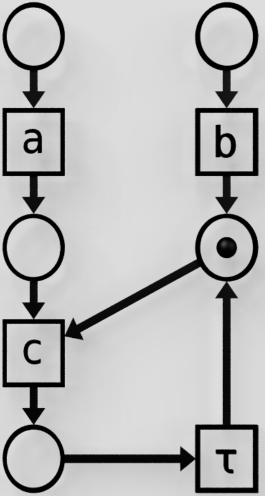


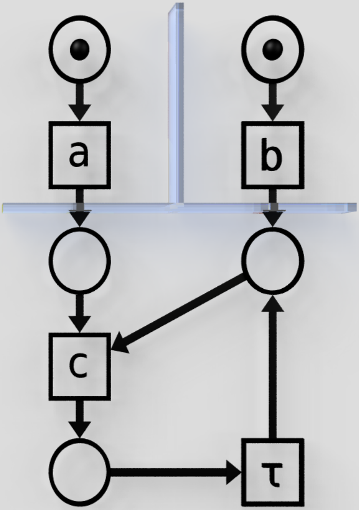
Distributed Nets

Let $N = (S, T, F, M_0, \ell)$ be a net.

An equivalence relation $\equiv_D \subseteq (S \cup T) \times (S \cup T)$ is a *distribution* iff

- $\forall t \in T, s \in {}^\bullet t. s \equiv_D t$, and
- if $M \in [M_0\rangle$ and $M[\{t, u\}\rangle M'$ then $s \not\equiv_D t$.

N is *distributed* if any distribution exists.



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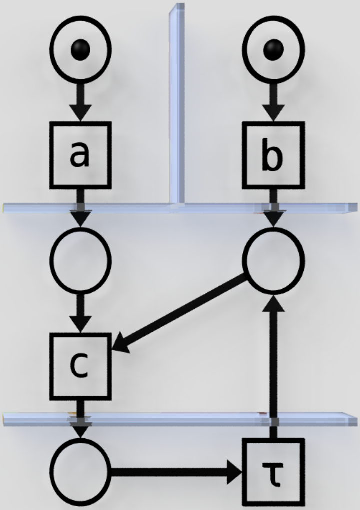
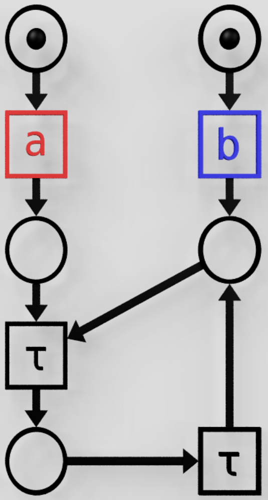


Firing Rule with Causality

- Non-τ transitions denote interactions with the environment.
- Tokens remember causal history.
- When recording only set of historic labels, statespace becomes finite for finite nets.



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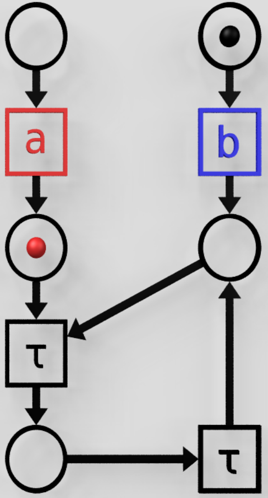

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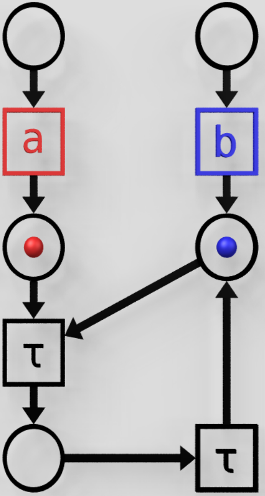

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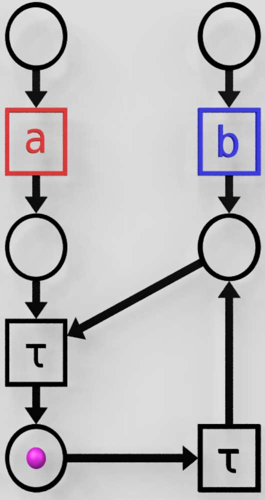

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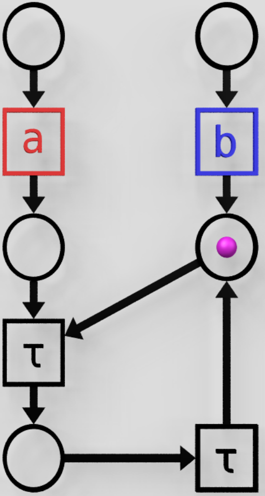


Processes

A pair $\mathcal{P} = (\mathcal{N}, \pi)$ is a *process* of a net $N = (S, T, F, M_0, \ell)$ iff

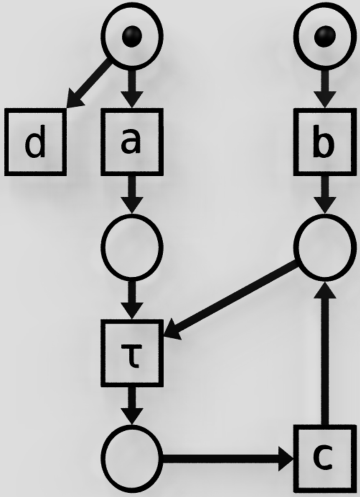
- $\mathcal{N} = (\mathcal{S}, \mathcal{T}, \mathcal{F}, \mathcal{M}_0, \mathcal{l})$ is a net, satisfying
  - $\forall s \in \mathcal{S}. |{}^\bullet s| \leq 1 \geq |s^\bullet| \wedge \mathcal{M}_0(s) = \begin{cases} 1 & \text{iff } {}^\bullet s = \emptyset \\ 0 & \text{otherwise} \end{cases},$
  - all arc-weights are 1, i. e. $\mathcal{F}(x, y) \in \{0, 1\}$ for all x, y and $\mathcal{F}$ can be considered a relation,
  - $\mathcal{F}$ is acyclic, i. e. $\forall x \in \mathcal{S} \cup \mathcal{T}. (x, x) \notin \mathcal{F}^+$, where $\mathcal{F}^+$ is the transitive closure of $\mathcal{F}$,
  - and $\{t \mid (t, u) \in \mathcal{F}^+\}$ is finite for all $u \in \mathcal{T}$.
- $\pi : \mathcal{S} \cup \mathcal{T} \rightarrow S \cup T$ is a function with $\pi(\mathcal{S}) \subseteq S$ and $\pi(\mathcal{T}) \subseteq T$, satisfying
  - $|\pi^{-1}(s) \cap \mathcal{M}_0| = M_0(s)$ for all $s \in S$,
  - $\forall t \in \mathcal{T}, s \in S.\ F(s, \pi(t)) = |\pi^{-1}(s) \cap {}^\bullet t| \wedge F(\pi(t), s) = |\pi^{-1}(s) \cap t^\bullet|$, and
  - $\forall t \in \mathcal{T}. \mathcal{l}(t) = \ell(\pi(t))$.

Processes

Let $\mathcal{P} = (\mathcal{N}, \pi)$ be a process and $\mathcal{N} = (\mathcal{S}, \mathcal{T}, \mathcal{F}, \mathcal{M}_0, \ell)$.

- The *end of the net* $\mathcal{N}^\circ$ is the set $\{s \in \mathcal{S} \mid s^\bullet = \emptyset\}$.
- $\mathcal{P}$ is *maximal* iff $\nexists G . \pi(\mathcal{N}^\circ)[G\rangle_N$.
- The set of all maximal processes of a net N is denoted by $MP(N)$.

Example: Processes of a Petri Net



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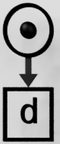

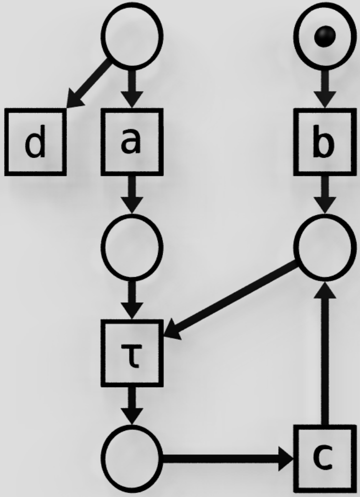

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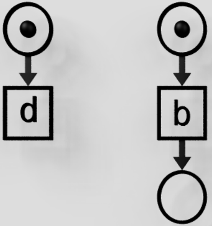

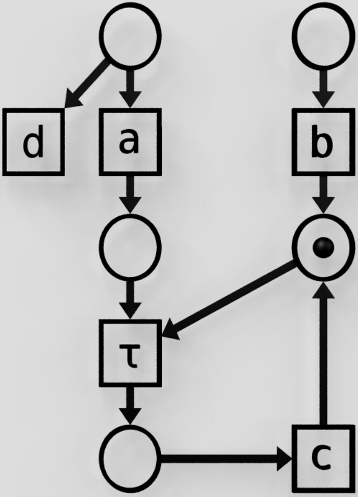

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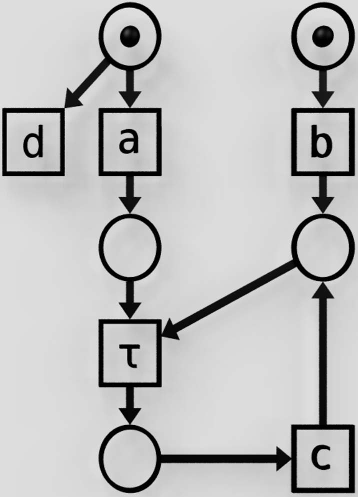

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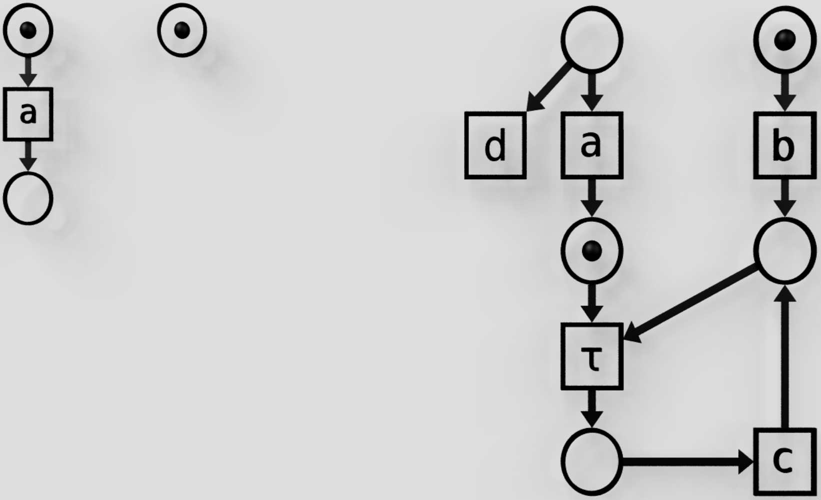

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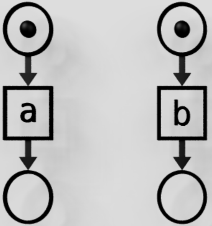

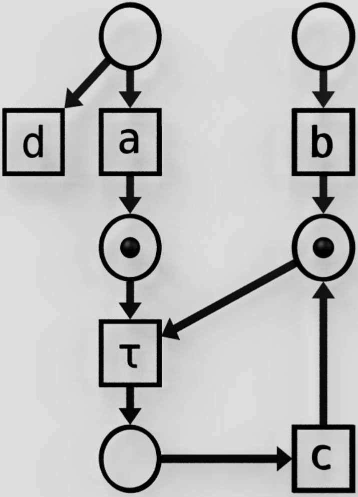

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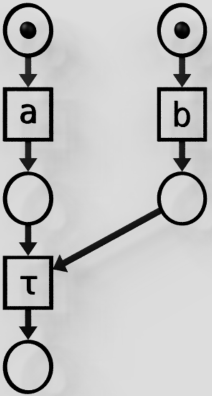

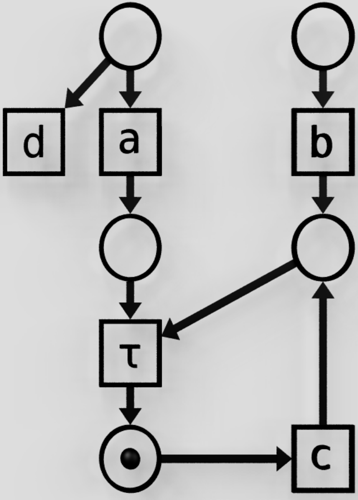

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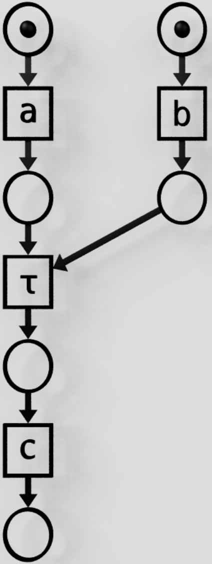

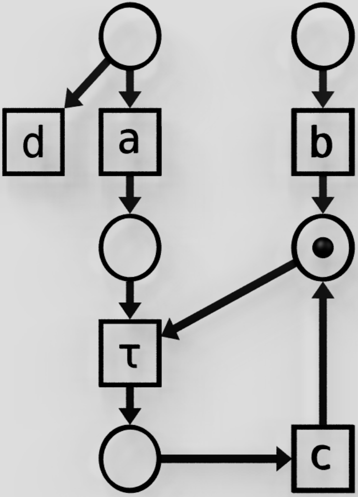

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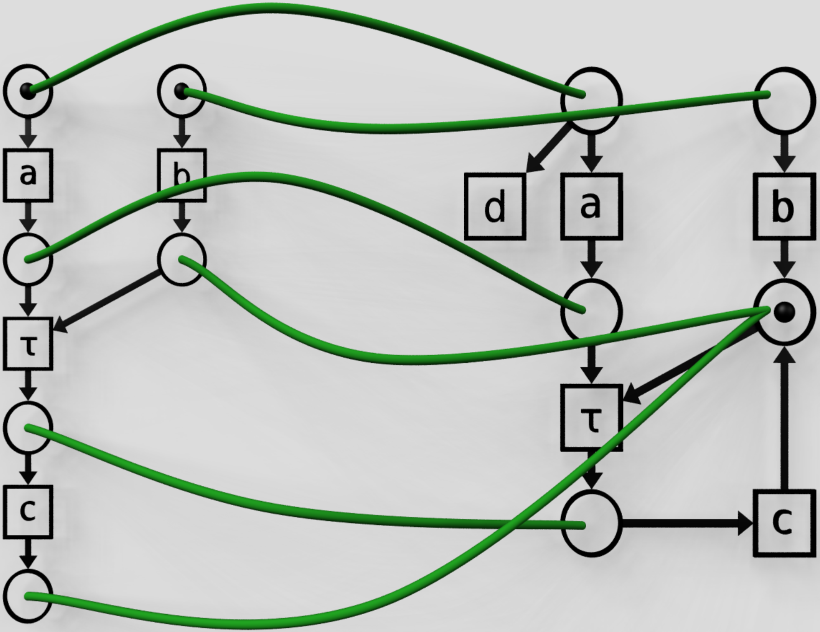


Labelled Partial Orders

A *labelled partial order* is a structure $(V, T, \leq, \ell)$ where

- V is a set of *vertices*,
- T is a set of *labels*,
- $\leq \subseteq V \times V$ is a partial order relation,
- $\ell : V \to T$ (the *labelling* function).

Pomsets

Two labelled partial orders $o = (V, T, \leq, \ell)$ and $o' = (V', T', \leq', \ell')$ are *isomorphic*, $o \approxeq o'$ iff there exists a bijection $\phi : V \to V'$ such that

- $\forall v \in V . \ell(v) = \ell'(\phi(v))$ and
- $\forall u, v \in V . u \leq v \Leftrightarrow \phi(u) \leq \phi(v)$.

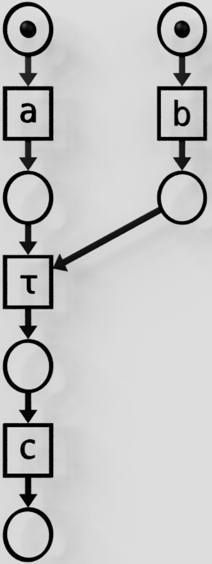
The *pomset* of o is its isomorphism class $[o] := \{o' \mid o \approxeq o'\}$.

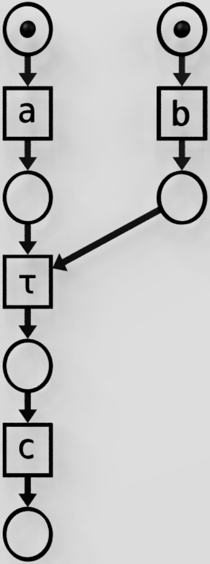
Pomset Traces

Let $\mathcal{P} = ((\mathcal{S}, \mathcal{T}, \mathcal{F}, \mathcal{M}_0, \ell), \pi)$ be a process.

- Let $\mathcal{O} := \{t \in \mathcal{T} \mid \ell(t) \neq \tau\}$, i.e. the visible transitions of the process.
- The *visible pomset* of $\mathcal{P}$ is the pomset $VP(\mathcal{P}) := [(\mathcal{O}, \mathrm{Act}, \mathcal{F}^* \cap \mathcal{O} \times \mathcal{O}, \ell \cap (\mathcal{O} \times \mathrm{Act}))]$ where $\mathcal{F}^*$ is the transitive and reflexive closure of the flow relation $\mathcal{F}$.
- $\mathrm{MVP}(N) := \{VP(\mathcal{P}) \mid \mathcal{P} \in MP(N)\}$ is the set of visible pomsets of all maximal processes of N.
- Two nets N and N' are *completed pomset trace equivalent*, $N \approx_{\mathcal{C}PT} N'$, iff $\mathrm{MVP}(N) = \mathrm{MVP}(N')$.

Example: Completed Pomset Trace





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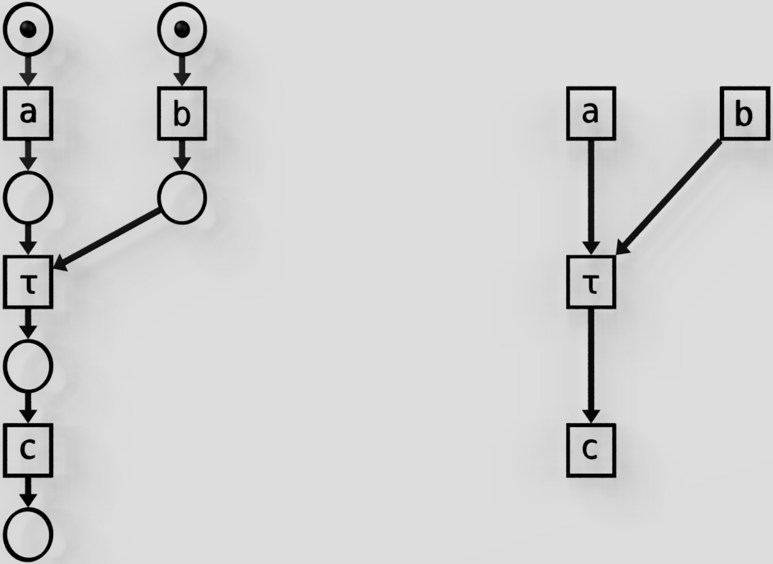

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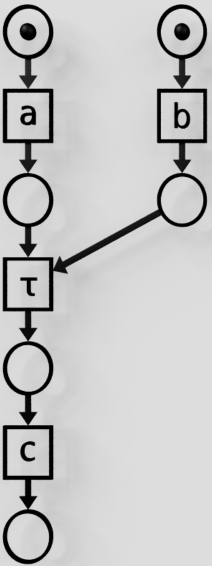

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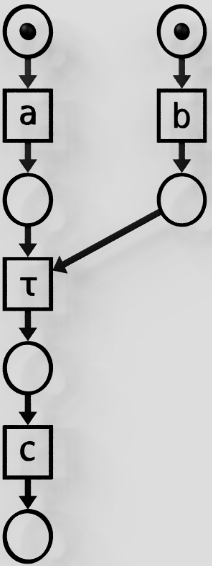
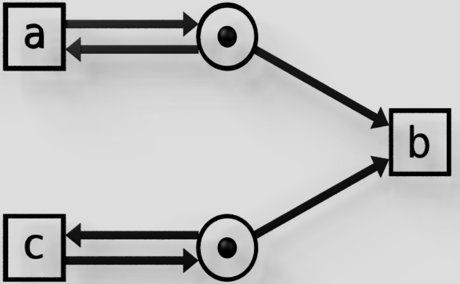


Completed Pomset Trace Equivalence

- Tracks causality
- Tracks deadlocks
- Tracks divergence
- Abstracts from transition identities
- Abstracts from decision structure
- Abstracts from non-diverging silent transitions

Formal Problem Statement



Can we find a 1-safe, finite, and distributed Petri net which is completed pomset trace equivalent to this specification net?

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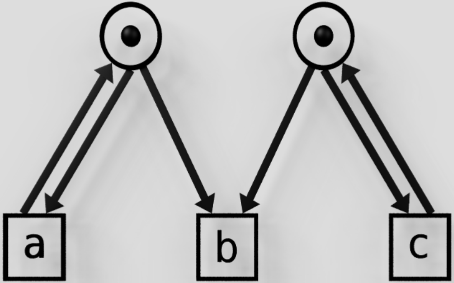

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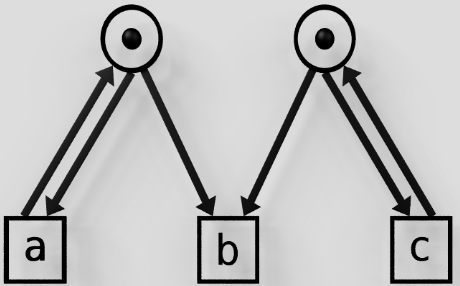


Can we find a 1-safe, finite, and distributed Petri net which is completed pomset trace equivalent to this specification net?

No.

Sketch of the Proof

- Track only token colour, not full history.
- Extend markings to *dependency markings* $M : (S \times 2^{\mathrm{Act}}) \to \mathbb{N}$.
- Finite 1-safe net has infinite runs, but statespace of size at most $m := (1 + 2^{|\mathrm{Act}|})^{|S|}$, i.e. finite.

Lemma: For a dependency marking M,
if $M[\{t_1\}\rangle[\{t_2\}\rangle \cdots [\{t_n\}\rangle M$

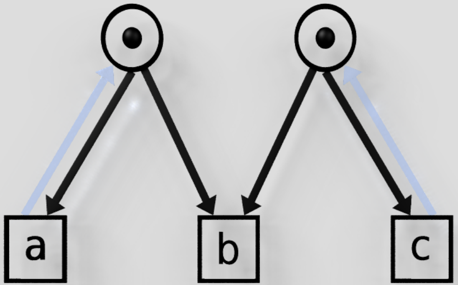
- all tokens produced by t_i have the same dependencies as those consumed,
- as otherwise, the less-dependent tokens could have been produced without t_i; violating 1-safety.

Sketch of the Proof

Theorem: There is no 1-safe, finite, distributed Petri net which is completed pomset trace equivalent to our specification.

- Specification can fire $(ac)^m b$.
- While doing so, some dependency marking M_l must be reached twice.
- With the Lemma, partition the loop into *a*-coloured and *c*-coloured part.
- While a^m can be fired, must also be able to fire c^m, otherwise new pomset with finitely many c but infinitely many a is generated. Dito with a and c reversed.
- In $(ac)^m b$ a single transition must have consumed an *a*-coloured and a *c*-coloured token, hence these two tokens reside on co-located places.
- As these tokens lead independently to a^m resp. c^m there are two concurrently firing transactions consuming them, hence they must be on different locations.

Core of the Problem



The “M” (i.e. optional coordination).

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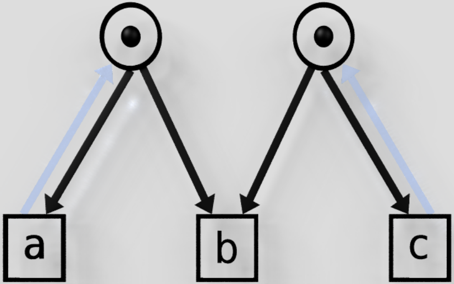


The “M” (i.e. optional coordination).

Is this particular to completed pomset trace equivalence?

Finite Step Failures Equivalence

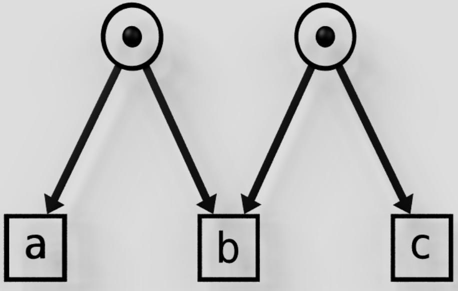
Whenever the net can only fire visible transitions, record a step failure pair, i. e.

- the trace of labels leading up to this marking, and
- all finite multisets of labels which can not fire in the next step.

Compare set of recorded step failure pairs.

- Abstracts from causality
- Tracks deadlocks
- Tracks divergence
- Abstracts from transition identities
- Tracks decision structure
- Abstracts from non-diverging silent transitions
- Tracks concurrency

Counterexample for Finite Step Failures Equivalence



Coarser than Finite Step Failures?

Without branching structure (“linear–time”):

- Decide everything on central location, execute visible transitions on distributed locations

With interleaving semantics:

- Connect all transitions to central scheduling place

When allowing divergence:

- Busy-wait all decisions

Finer than Finite Step Failures?

- Results stable up to branching ST-bisimilarity with explicit divergence.
- Includes practically the entire branching-time part of Rob's spectrum.

Structures Smaller than M?

- Answer comprises the largest part of my thesis
- . . . because everything non-M can be implemented

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- ...because everything non-M can be implemented
- About 10 pages to describe the necessary Petri net construction
- About 30 pages for the correctness proof (with huge invariant)
- About 8 pages to describe a Petri net to distributed C compiler used for testing
- ...and finding a bug in the already published construction and proof

What About Other Formal Models?

How dependent are the results on the choice of Petri nets specifically?

- Not terribly so: π-calculus results co-developed by Peters & Nestmann, formal connection was being worked on by Mennicke.
- Some hardware-centric results by Lamport seem related, no formal connection established.

Ways to Evade the Negative Theorems

- Don't use branching time and solve the consensus problems probabilistically.
- Assume bounded message delays (often needed for error detection anyway).
- Use approximately uniform passage of time.
- Physical effects not accurately captured by Petri nets.

Open Problems and Questions

- Where between weak completed step trace equivalence and finite step failures equivalence become Ms unimplementable?
- Conjecture: There is some “asynchronous branching time” equivalence (and Ms are implementable therein).
- Which structure(s) delineate(s) the limit of distributed implementability when checking all three of divergence, causality and branching time?
- Stability of Ms in non-safe nets under causality only conjectured so far.
- Efficient modelling of quantum-mechanical effects for distributed computing.

References and Earlier Publications

The thesis includes content from various papers with Glabbeek, Goltz, Mennicke, Nestmann, Peters (alphabetically ordered).

“External” must-reads:

- Best and Darondeau: “Petri net distributability”
- Gorla: “On the relative expressive power of asynchronous communication primitives”
- Hopkins: “Distributable nets”
- Palamidessi: “Comparing the expressive power of the synchronous and the asynchronous π-calculus”
- Taubner: “Zur verteilten Implementierung von Petrinetzen”

Results

- Identified the M as a problematic structure for distributed implementations
- Showed stability under causality respecting equivalences
- Showed stability under branching time equivalences
- Showed the M to be the smallest such structure, by concrete implementation for all other cases
- Showed an infinite hierarchy of bigger problematic structures exists
- Established formal connections between free-choice Petri nets and asynchronous nets (omitted in this talk)
- Described LSGA-nets as an alternative and equivalent approach to generate distributed nets (dito)
- Described structural conflict nets and showed them to be a class of nets the implementation is valid for (dito)

Thank You!

- Institut für Programmierung und Reaktive Systeme @ TU Braunschweig
- National ICT Australia
- Deutsche Forschungsgemeinschaft
- Deutscher Akademischer Austauschdienst
- Studienstiftung des deutschen Volkes